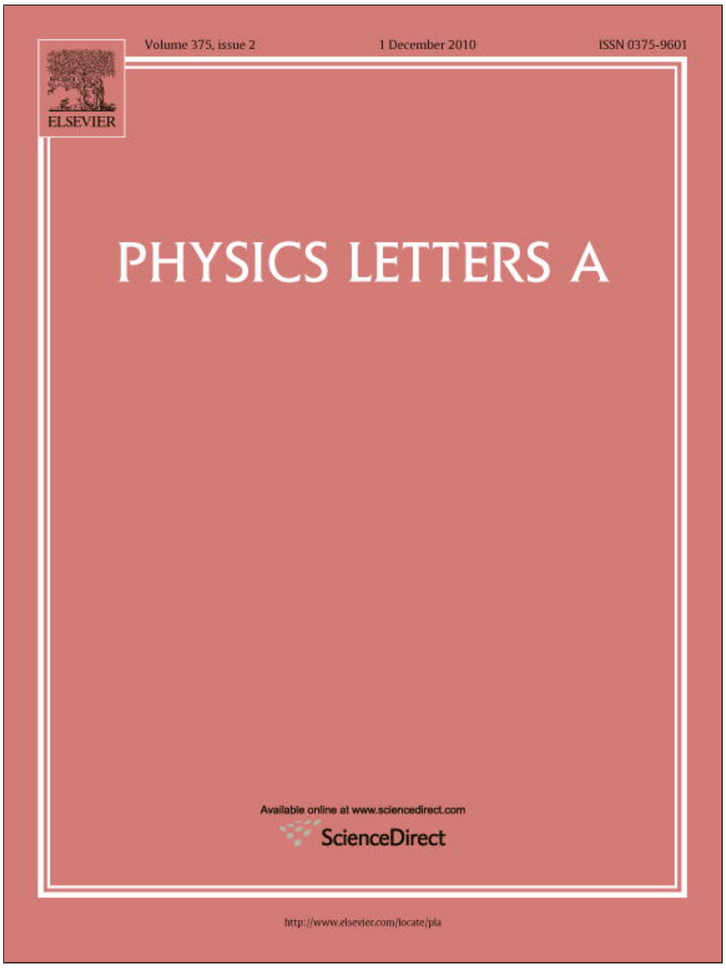
Volume 375, issue 2
1 December 2010
ISSN 0375-9601
ELSEVIER
PHYSICS LETTERS A
Available online at www.sciencedirect.com
ScienceDirect
http://www.elsevier.com/locate/pla

Physics Letters A 375 (2010) 220–224

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Characterization of the variation of the material properties in a freestanding inhomogeneous thin film

Xiaoshan Cao [a], Feng Jin [b], Insu Jeon [a,*]

[a] School of Mechanical Systems Engineering, Chonnam National University, 300 Yongbong-dong, Buk-gu, Gwangju 500-757, Republic of Korea
[b] MOE Key Laboratory for Strength and Vibration, School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China

ARTICLE INFO

Article history:
Received 6 October 2010
Received in revised form 30 October 2010
Accepted 2 November 2010
Available online 5 November 2010
Communicated by R. Wu

Keywords:
Lamb wave
Functionally graded material (FGM)
Cut-off frequency
Wentzal–Kramers–Brillouin (WKB) method
Material property variation

ABSTRACT

This Letter presents a new technique for measuring the variation of the material properties along the thickness in a freestanding inhomogeneous thin film. The analytical results reveal a simple relation between the material properties and the set of cut-off frequencies of Lamb waves. The influence of the graded properties on the variation of cut-off frequencies in three different kinds of models, including artificial FGM model, sub-surface damage model, and nano-porous thin film model, is discussed. These results provide theoretical guidance for characterizing the material property variations of MEMS/NEMS.

1. Introduction

Since the concept of functionally graded material (FGM) was first proposed in the 1980s [1], this type of material has attracted considerable attention from researchers in diverse scientific and engineering fields, such as electronics, optics, biology, chemistry, biomedical engineering, nuclear engineering, mechanical engineering, and so on [2]. With the development of material technology, artificial FGMs are manufactured and used in micro-/nano-electromechanical systems (MEMS/NEMS) for achieving high sensitivity and good performance [3–5].

Furthermore, natural FGMs have already existed in MEMS/NEMS. The natural FGMs form in a nano-structure when they undergo surface erosion, surface aging, or sub-surface damage. The material properties of the thin film around the surface, named the sub-surface region, should vary along its thickness [6–8]. Since both artificial and natural FGMs might exist, an understanding of the material properties of FGM thin film is of fundamental importance in designing and evaluating the performance of MEMS/NEMS.

To detect the material property variations in the sub-surface region, the guided wave technique has been widely used. Most studies have focused on the dispersion properties influenced by the variation of material properties. Flannery et al. [6] have studied the measurement of the porosity and stiffness of nano-porous aerogel films through wideband ultrasonic surface waves. Paehler et al. [9] have investigated the characterization of sub-surface damage in silicon wafers through laser acoustics. Cao et al. [7] have put forward a functionally graded piezoelectric layered-structure model and discussed the influence of the sub-surface region on the propagation of surface waves. In these studies, the thickness of the wafer is much larger than that of the sub-surface region so that the wafer can be assumed as an infinite half space. However, the assumption is not appropriate when the thickness of the sub-surface region is only slightly smaller than that of the thin film. We should use Lamb waves for detecting the mechanical properties in such thin film.

Many reports have been published on the behavior of Lamb waves in various homogeneous or inhomogeneous plates [10,11]. Most of these studies have focused on analyzing the dispersion curves and wave structures. Being one of the important properties of Lamb waves, the cut-off frequencies in homogeneous plates also have been investigated by some scientists. Such studies always have been focused on corrosion detection and thickness measurement in a variety of structures [10,12]. However, no reports have been published on the relations between the cut-off frequencies of Lamb waves and material properties in an inhomogeneous free-standing thin film.

In this Letter, we consider both an artificial and a natural FGM thin film. Using the Wentzal–Kramers–Brillouin (WKB) method

* Corresponding author. Tel.: +82 62 530 1688; fax: +82 62 530 1689.
E-mail address: i_jeon@chonnam.ac.kr (I. Jeon).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2010.11.004

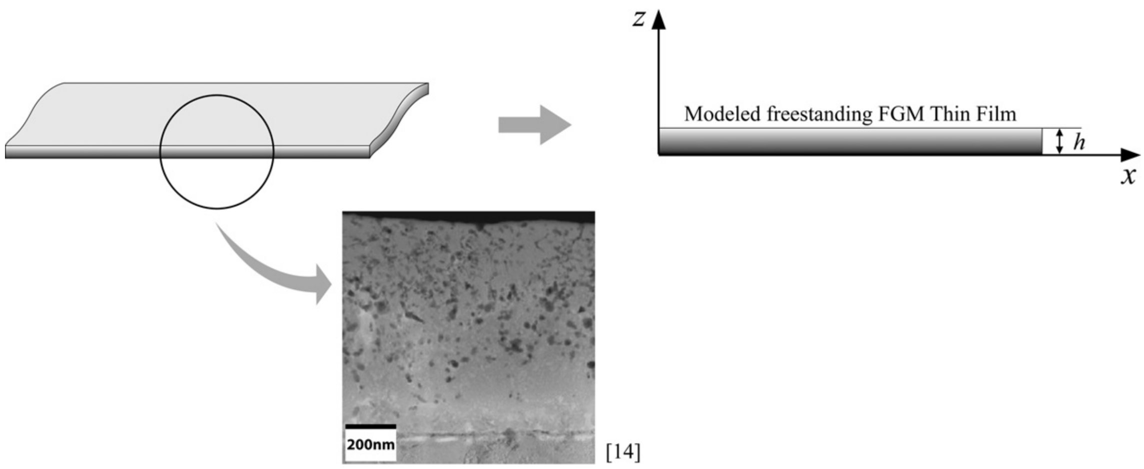

Fig. 1. The modeling of a freestanding FGM thin film structure.

[13], we obtain the general equations for the relation between the gradient parameters of material properties and cut-off frequencies. Through these equations, we also investigate the sensitivity of the cut-off frequencies to the gradient parameter in three different models.

2. Statement of the problem and governing equations

Consider a freestanding thin film that is made of an artificial or natural FGM, and that Lamb waves propagate along the x direction in the thin film, as shown in Fig. 1 [14]. It is assumed that the material properties of the FGM vary continuously along the thickness (the z-axis), i.e., all of the properties, such as the elastic coefficients and density, are functions of the z-axis. The motion is restricted in the xoz plane and the Lamb waves propagate in the positive direction of the x-axis. The constitutive equations can be expressed as follows:

$$\sigma_{ij} = c_{ijkl} S_{kl}. \tag{1}$$

In Eq. (1), σ_{ij} and S_{kl} are the stress and strain tensors, and c_{ijkl} are the elastic coefficients. The motion equations are given by:

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{2}$$

where ρ is the density, u_i is the component of the mechanical displacement in the ith direction, and the dot, "˙" represents time differentiation. The relation between the mechanical displacements and the stain components are as follows:

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

Owing to the assumption of plane stain, the displacement components can be described as:

$$u = u(x, z, t), \qquad v = 0 \quad \text{and} \quad w = w(x, z, t). \tag{4}$$

For brevity, $\exp[ik(x - ct)]$ is omitted in this Letter, where k is the wave number, c is the phase velocity, and $i = \sqrt{-1}$. From Eqs. (1)–(4), the governing equations in an FGM thin film are rewritten as follows:

$$c_{44}u'' + c'_{44}u' + (\rho\omega^2 - c_{11}k^2)u + (c_{13} + c_{44})ikw' + ikc'_{44}w = 0, \tag{5}$$

$$c_{11}w'' + c'_{11}w' + (\rho\omega^2 - c_{44}k^2)w + (c_{44} + c_{13})iku' + c'_{13}iku = 0. \tag{6}$$

Here, the symbols, "′" and "″" represent the first and second differentials with respect to z.

When a Lamb wave propagates in the thin film, the boundary conditions must be satisfied. The traction free condition is: at $z = 0, h$,

$$\sigma_z = 0, \qquad \sigma_{xz} = 0. \tag{7}$$

To study the cut-off frequencies, we take into account the limiting condition of $k \to 0$. In this case, the governing equations can be simplified as follows:

$$(c_{44}u')' + \rho\omega_n^2 u = 0, \tag{8}$$

$$(c_{11}w')' + \rho\omega_n^2 w = 0. \tag{9}$$

In the above, ω_n is the cut-off frequency. Therefore, the problem of the cut-off frequencies of Lamb waves in a freestanding FGM thin film is simplified as the problem in which the variables satisfy the governing equations including Eqs. (8) and (9) under the traction free boundary conditions, as shown in Eq. (7).

3. Solution of the problem

To solve the ordinary differential equation with variable coefficients, we use the WKB method [13,15].

$$u = \frac{a_1 \cos(\int_0^z \sqrt{\rho/c_{44}}\, dz\, \omega_n) + a_2 \sin(\int_0^z \sqrt{\rho/c_{44}}\, dz\, \omega_n)}{(\rho c_{44})^{1/4}}. \tag{10}$$

Similarly, we have:

$$w = \frac{b_1 \cos(\int_0^z \sqrt{\rho/c_{11}}\, dz\, \omega_n) + b_2 \sin(\int_0^z \sqrt{\rho/c_{11}}\, dz\, \omega_n)}{(\rho c_{11})^{1/4}}. \tag{11}$$

Substituting Eqs. (10) and (11) into the traction free conditions, we obtain a set of homogeneous linear algebraic equations for determining a_1, a_2, b_1, and b_2. The sufficient and necessary condition for the existence of a non-trivial solution is that the determinant of the coefficient matrix has to vanish. Consequently, we have the following.

$$\tan\left(\int_0^h \sqrt{\rho/c_{44}}\,\omega_{nT}\, dz\right) = \frac{A(0)A'(h)\sqrt{\rho_0/c_{44_0}} - A'(0)A(h)\sqrt{\rho_h/c_{44_h}}}{A'(0)A'(h) + A(0)A(h)\omega_{nT}^2\sqrt{\rho_0\rho_h/c_{44_0}c_{44_h}}}\,\omega_{nT}, \tag{12}$$

$$\tan\left(\int_0^h \sqrt{\rho/c_{11}}\,\omega_{nL}\,dz\right) = \frac{B(0)B'(h)\sqrt{\rho_0/c_{110}} - B'(0)B(h)\sqrt{\rho_h/c_{11h}}}{B'(0)B'(h) + B(0)B(h)\omega_{nL}^2\sqrt{\rho_0\rho_h/c_{110}c_{11h}}}\,\omega_{nL}. \qquad (13)$$

Here, $A(z) = (\rho c_{44})^{-1/4}$ and $B(z) = (\rho c_{11})^{-1/4}$, respectively, and ω_{nT} and ω_{nL} are cut-off frequencies which satisfy Eq. (12) or (13), respectively. ω_{nT} is a function related to only the density and the elastic coefficient c_{44}, while ω_{nL} is a function related to only the density and the elastic coefficient c_{11}.

Furthermore, in Eqs. (12) and (13), when the material properties slowly vary along the thickness and ω_{nT} and ω_{nL} are large value, the right-hand side of the equation approaches zero. Hence, we have $\int_0^h \sqrt{\rho/c_{44}}\,\omega_{nT} \to n\pi$, i.e., $\omega_{nT} \to n\pi/\int_0^h \sqrt{\rho/c_{44}}$. Similarly, it can be deduced that $\int_0^h \sqrt{\rho/c_{11}}\,\omega_{nL} \to n\pi$ from Eq. (13), i.e., $\omega_{nL} \to n\pi/\int_0^h \sqrt{\rho/c_{11}}$. We define:

$$D(\omega_{nT}) = \omega_{(n+1)T} - \omega_{nT} \quad \text{and} \quad D(\omega_{nL}) = \omega_{(n+1)L} - \omega_{nL}.$$

Consequently, as n increases, we have the following.

$$D(\omega_{nT}) \to \pi \Big/ \int_0^h \sqrt{\rho/c_{44}}\,dz, \qquad (14)$$

$$D(\omega_{nL}) \to \pi \Big/ \int_0^h \sqrt{\rho/c_{11}}\,dz. \qquad (15)$$

From Eqs. (14) and (15), we can consider the set of cut-off frequencies to be a union of two series of approximate arithmetic progressions, when the material parameters are continuous and derivable with respect to the thickness.

4. Numerical results and discussion

Considering the scientific application, we suppose three different kinds of possible model:

4.1. Artificial FGM model

An artificial FGM thin film is always composed of two different kinds of material, while the volume percentage of each material varies along the thickness [16]. The form of the material parameters is:

$$\alpha = \alpha_1 f(z) + \alpha_2\big[1 - f(z)\big], \qquad (16)$$

where α indicates an arbitrary material parameter, and $f(z)$ and $1 - f(z)$ represent the volume percentage of materials 1 and 2, respectively. This model is fit for not only an artificial FGM thin film, but also a degraded thin film under surface erosion because it can be considered as composed of an original material and a material formed by reaction.

4.2. Sub-surface damage model

A few scientists have investigated surface waves in a silicon wafer or a piezoelectric wafer with sub-surface damage [7,9]. Some of them consider the wafer as a layered structure with a homogeneous substrate covered by many different homogeneous sublayers, while others consider it as an FGM layered structure, which is a homogeneous substrate covered by an FGM layer. In their reports, the variation of the density is so slight that it can be disregarded. Similarly, in some conditions, the variation of elastic coefficients is much more obvious than that of the density. We can suppose that the material properties vary along the thickness as follows:

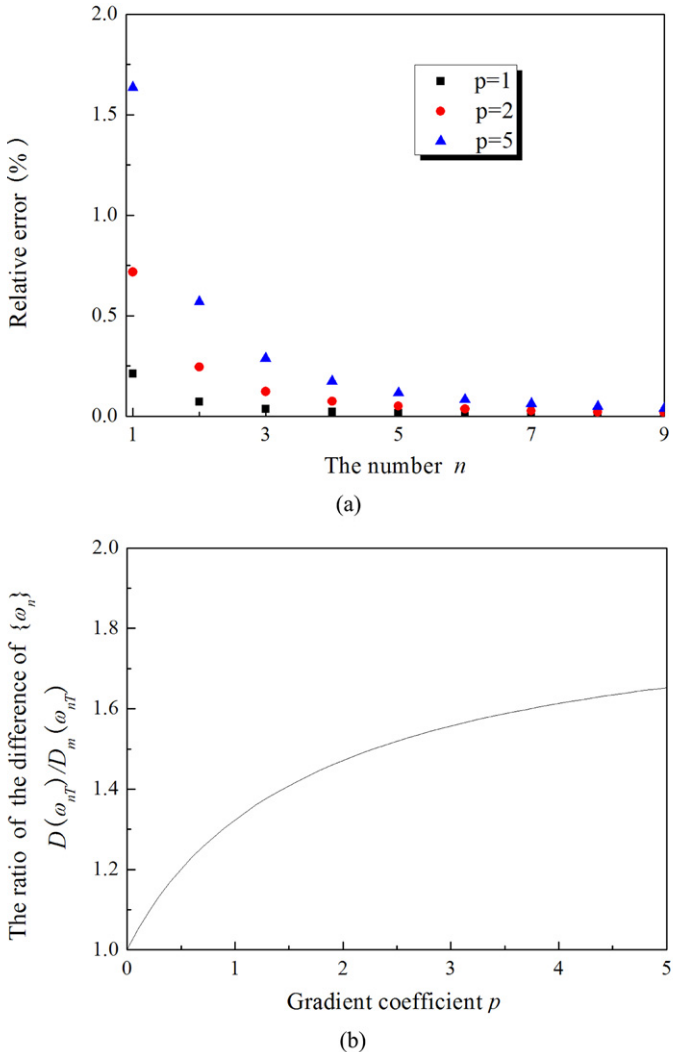

Fig. 2. The influence of the gradient parameter on the cut-off frequency in Model I. (a) The relative error between $\pi/\int_0^h \sqrt{\rho/c_{44}}\,dz$ and $\omega_{(n+1)T} - \omega_{nT}$. (b) The relation between $D(\omega_{nT})$ and the gradient parameter.

$$\rho = \text{const}, \qquad c_{11} = c_{11}(z), \qquad c_{44} = c_{44}(z).$$

4.3. Nano-porous material model

It is well known that the Young's modulus, E, in porous material follows the power-law dependence [6]. We suppose that the density varies along the thickness and that the elastic coefficients are a function of the density as follows:

$$\rho = \rho(z), \qquad c_{11}/c_{110} = (\rho/\rho_0)^m, \qquad c_{44}/c_{440} = (\rho/\rho_0)^m.$$

These three models describe three conditions: the variation of the density and the elastic coefficients with the volume percentages; the variation of only the elastic coefficients along the thickness; and the variation of the elastic coefficients with the density. The following numerical results are focused on the influence of the gradient parameter on the cut-off frequencies.

Because the forms of Eqs. (14) and (15) are similar, we discuss only ω_{nT} and $D(\omega_{nT})$ in the numerical results. Suppose the FGM thin film in Model I is composed of a kind of metal and ceramic with the following parameters.

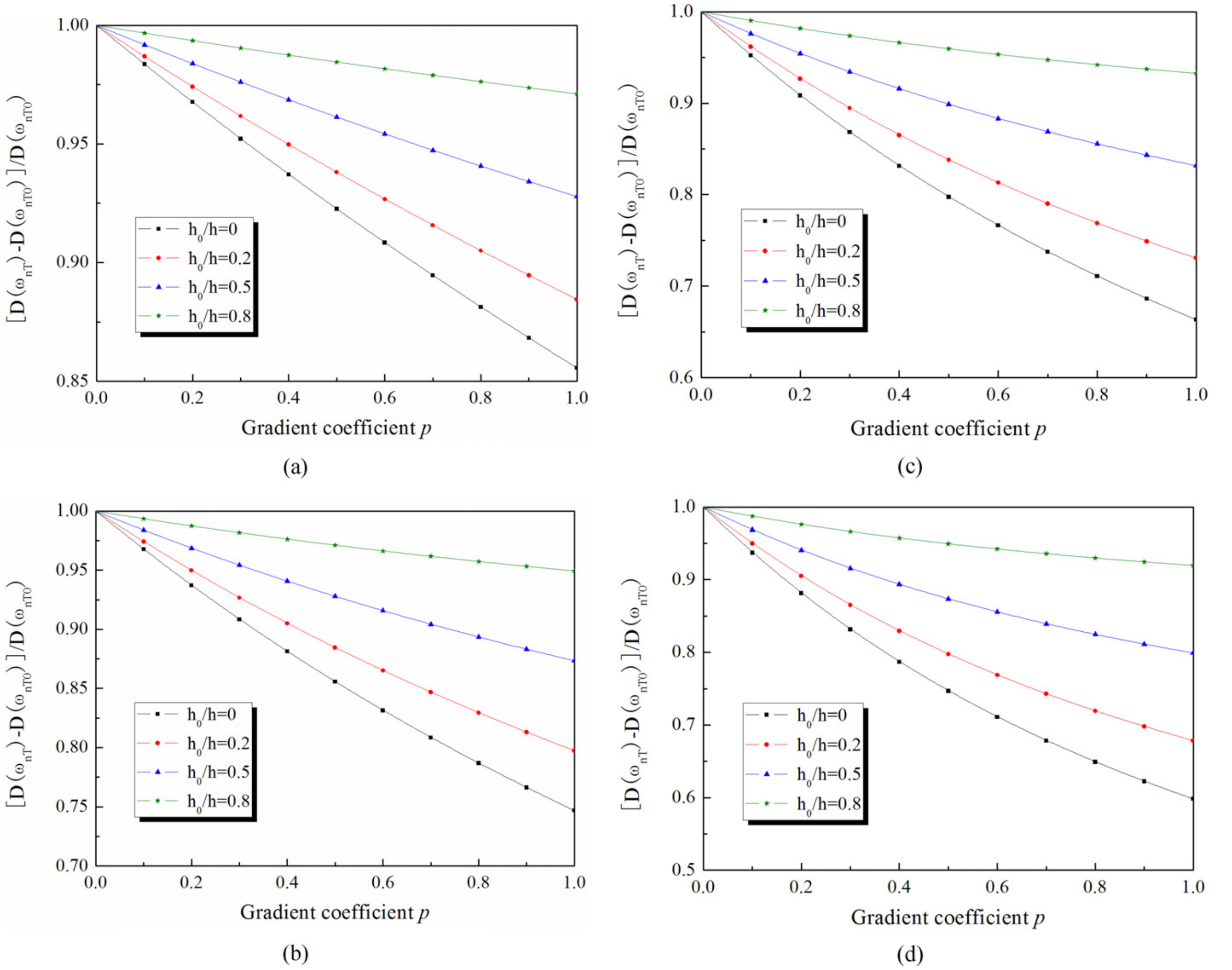

Fig. 3. The influence of the gradient parameter on the cut-off frequencies in Model II and Model III. (a) Model II and Model III ($m = 2$), (b) Model III ($m = 3$), (c) Model III ($m = 4$) and (d) Model III ($m = 5$).

Ni: $c_{44m} = 76$ GPa and $\rho_m = 8902\ \text{kg/m}^3$;

Ceramic: $c_{44c} = 118.11$ GPa and $\rho_c = 3900\ \text{kg/m}^3$;

$\alpha = \alpha_m (z/h)^p + \alpha_c[1 - (z/h)^p]$.

In the above, α, α_m, and α_c indicate the material parameter of FGM, metal, and ceramic, respectively, and p is the gradient parameter. Fig. 2(a) plots the relative error, viz., the ratio of $\pi/\int_0^h \sqrt{\rho/c_{44}}\,dz - (\omega_{(n+1)T} - \omega_{nT})$ to $\pi/\int_0^h \sqrt{\rho/c_{44}}\,dz$, where $\omega_{(n+1)T} - \omega_{nT}$ is obtained by Eq. (12). The error is too slight and becomes slighter as n increases. When n is larger than 10, the relative error is less than 0.05%. This means that the measurement of material properties through cut-off frequencies is effective and reasonable because $\pi/\int_0^h \sqrt{\rho/c_{44}}\,dz$ is the limit of $D(\omega_{nT})$. Using these results, we find that the limit of $D(\omega_{nT})$ increases as the gradient parameter, p, increases in this thin film, as shown in Fig. 2(b), where $D_m(\omega_{nT})$ denotes the difference of cut-off frequency of Lamb waves in homogeneous metal plate, namely $D_m(\omega_{nT}) = \pi\sqrt{c_{44m}/\rho_m}/h$. When $p = 0$, the plate is homogeneous metal plate, so that $D(\omega_{nT})/D_m(\omega_{nT}) = 1$. The relation between $D(\omega_{nT})$ and the gradient parameter will give us a new method for characterizing the changes in material properties by using the cut-off frequencies.

To compare the sensitivity of $D(\omega_{nT})$ to the gradient parameter, we select the material parameters of Model II and Model III as follows:

Model II.

$$\rho = \rho_0,$$

$$c_{44}(z) = \begin{cases} c_{440}\exp[-p(z-h_0)^2/(h-h_0)^2] & h_0 < z \leqslant h, \\ c_{440} & 0 < z \leqslant h_0. \end{cases}$$

Model III.

$$\rho(z) = \begin{cases} \rho_0\exp[-p(z-h_0)^2/(h-h_0)^2] & h_0 < z \leqslant h, \\ \rho_0 & 0 < z \leqslant h_0. \end{cases}$$

In the above, p is the gradient parameter and $h_0 < z \leqslant h$ indicates the sub-surface region. Fig. 3 plots the relation between $D(\omega_{nT}) - D(\omega_{nT0})$, which is the variation of $D(\omega_{nT})$ due to the inhomogeneous property, the thickness of the sub-surface region, h_0, and the gradient parameter, p, where $D(\omega_{nT0})$ refers to the value of $D(\omega_{nT})$ in a homogeneous thin film with density ρ_0, elastic coefficient c_{440}, and thickness h. $D(\omega_{nT0})$ can be obtained by Eq. (14) and satisfies $D(\omega_{nT0}) = \pi c_T/h$, where $c_T = \sqrt{c_{44}/\rho}$ is the bulk shear wave velocity in homogeneous material. The same result on homogeneous thin film also can be obtained by reference [10]. For a certain model with the same thickness, $D(\omega_{nT})$ varies linearly with the gradient coefficient. The thicker the sub-surface region is, the more obvious the variation of $D(\omega_{nT})$ is. With the same gradient coefficient and thickness of the sub-surface region, the variation of $D(\omega_{nT})$ in Model II is the same as that in Model III with $m = 2$, as shown in Fig. 3(a), and the bigger m is, the more obvious the variation of $D(\omega_{nT})$ is in Model III, as shown in Figs. 3(a)–(d). These results suggest that it is possible to detect material parameters through the cut-off frequencies of Lamb waves if the sub-surface region is thick enough compared with the film thickness.

5. Conclusions

In conclusion, when the variation of material properties is continuous and derivable, the cut-off frequencies of Lamb waves in these freestanding inhomogeneous thin films can be decided by two simple equations. The set of cut-off frequencies is a union of two series of approximate arithmetic progressions. Here, the difference for one series varies inversely as the definite integral of $\sqrt{\rho/c_{44}}$ along the thickness and the other varies inversely as the definite integral of $\sqrt{\rho/c_{11}}$. The results reveal a simple and universal relation between material properties and cut-off frequencies and provide theoretical guidance for evaluating the variation of material properties in MEMS and NEMS.

Acknowledgement

This work was supported by Mid-career Researcher Program through NRF grant funded by the MEST (No. 2009-0083774).

References

[1] M. Koizumi, in: J.B. Holt, M. Koizumi, T. Hirai, Z. Munir (Eds.), Ceramic Transaction, Functionally Gradient Materials, American Ceramic Society, Westerville, 1993, pp. 3–10.
[2] C.F. Lu, C.W. Lim, W.Q. Chen, Int. J. Solids Struct. 46 (2009) 1176.
[3] Y. Fu, H. Du, W. Huang, S. Zhang, M. Hu, Sens. Actuators A: Phys. 112 (2004) 395.
[4] Y. Fu, H. Du, S. Zhang, Mater. Lett. 57 (2003) 2995.
[5] A.S. Mahmud, Y. Liu, T.H. Nam, Smart Mater. Struct. 17 (2008) 015031.
[6] C.M. Flannery, C. Murray, I. Streiter, S.E. Schulz, Thin Solid Films 388 (2001) 1.
[7] X. Cao, F. Jin, I. Jeon, Appl. Phys. Lett. 95 (2009) 261906.
[8] L.G. Zhou, H. Huang, Appl. Phys. Lett. 84 (2004) 1940.
[9] D. Paehler, D. Schneider, M. Herben, Microelectron. Eng. 84 (2007) 340.
[10] J.L. Rose, Ultrasonic Waves in Solid Media, Cambridge University Press, Cambridge, 1999.
[11] G.R. Liu, J. Tani, T. Ohyoshi, Trans. Jpn. Soc. Mech. Eng. 57 (1991) 131.
[12] J.L. Rose, D. Jiao, S.P. Pelts, J.N. Barshinger, M.J. Quarry, in: NACE International Corrosion 97, New Orleans, 1997, Paper No. 292.
[13] F.W.J. Olver, Asymptotics and Special Functions, Academic Press, New York, 1974.
[14] North Carolina State University, http://nanopatentsandinnovations.blogspot.com/2010/02/smart-nano-coating-opens-door-to-safer.html.
[15] X.S. Cao, F. Jin, Z.K. Wang, Acta Mech. 200 (2008) 247.
[16] L.S. Ma, T.J. Wang, Int. J. Solids Struct. 40 (2003) 3311.