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Stress intensities at the triple junction of a multilevel thin film package

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Abstract

Stress intensities of a singular near-tip field around the vertex of a triple junction wedge in multilevel thin film packages are calculated using the two-state M-integral. For this calculation the existence of a simple auxiliary field in the sense of the M-integral associated with every eigenfunction for the triple junction vertex is first verified numerically. The auxiliary field is then employed for superposition with the elastic field under consideration, and the associated two-state M-integral is computed via the domain integral technique. This enables us to extract the stress intensity of each singular eigenfunction for the triple junction vertex at three different junction angles, $\alpha = 45°$, 90° and 135°.

1. Introduction

Two-state conservation laws are frequently used for calculating the stress intensity of generic wedges including free edges, re-entrant edges and cracks at a bi-material interface. Chen and Shield [1] proposed three conservation laws for two equilibrium states, each law is derived from the well-known conservation integrals J, M and L for a single equilibrium state [2]. Among the three two-state conservation laws, the two-state J-integral has been widely employed for obtaining stress intensities and elastic T-stresses for cracks or for finding dislocation strengths [3–6]. The two-state L-integral has also been employed by Choi and Earmme [7] for determining the stress intensities of circular arc-shaped cracks.

The two-state M-integral was applied by Im and Kim [8] for computing the intensity of a singular near-tip field of a generic composite wedge. They proved the existence of the so-called conjugate eigenfunction in the sense of the M-integral for every eigenfunction at a generic wedge. This implies the existence of an auxiliary field in the form of the conjugate eigenfunction, which may be superposed with an elastic field under consideration. The two-state M-integral for the elastic field resulting from this superposition is employed for computing the intensity of a singular stress field of the original elastic field under consideration. Recently, Lee and Im [9] and Lee et al. [10] have shown that this two-state M-integral is efficiently utilized to obtain the near-tip intensities of three-dimensional wedges.

In this paper, we are concerned with applying the two-state M-integral to compute the stress intensities of a near-tip singular field for the triple junction wedge of three different isotropic materials under thermal loading. The present scheme gives us a simple and efficient way to compute the stress intensities at the triple junction vertex.

2. Governing equations and the two-state M-integral for a thermo-elastic field

Consider a triple junction wedge where three different elastic materials join one another and the junction angle α that represents the copper section takes (see Fig. 1). We assume that the three materials are isotropic and are rigidly joined to the neighboring material along a straight interface running radially from the junction vertex. Moreover,

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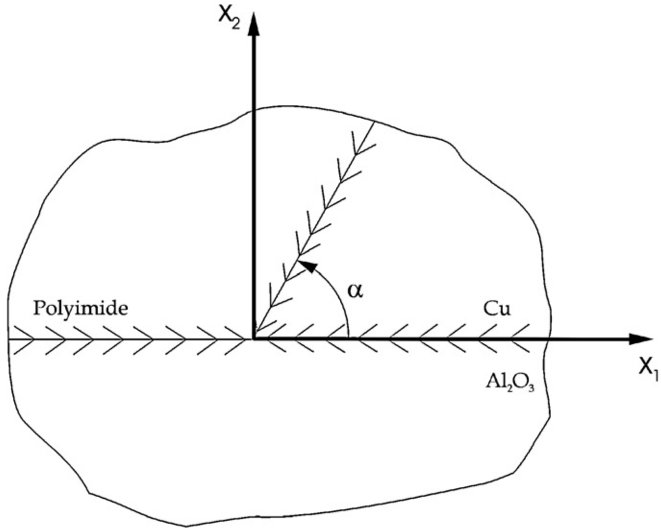

Fig. 1. The triple junction wedge of three different materials.

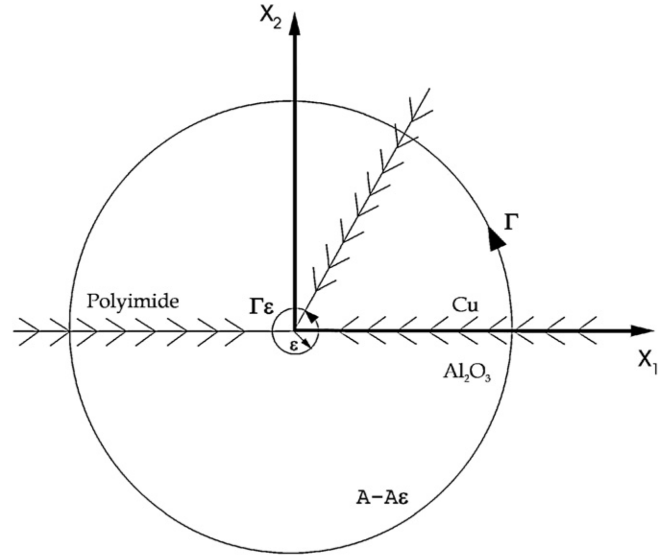

Fig. 2. The M-integral path and domain.

we restrict our attention to the plane strain problem where steady state thermal loading is prescribed.

Let σ_{ij} and ε_{ij} denote the Cartesian components of the stress and strain, respectively, and let T represents the temperature relative to the reference temperature. Suppose an isotropic body is in a state of plane strain deformation under quasi-static thermal loading. The governing equation for the temperature and elastic fields may be written as follows:

$$T_{,ii} = 0 \tag{1.a}$$

$$\sigma_{ij,j} = 0 \quad (i, j = 1, 2) \tag{1.b}$$

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2 \tag{1.c}$$

$$\sigma_{ij} = C_{ijkm}\varepsilon_{km} - \beta_{ij}T, \quad \beta_{ij} = \delta_{ij}E\alpha/(1-2v) \tag{1.d}$$

$$C_{ijkm} = \mu\delta_{ik}\delta_{jm} + \mu\delta_{im}\delta_{jk} + 2v\mu\delta_{ij}\delta_{km}/(1-2v) \tag{1.e}$$

where μ, v and α are the shear modulus, Poisson's ratio and thermal expansion coefficient, respectively, and 'i' indicates the partial differentiation with respect to the Cartesian coordinate x_i. The M-integral, in the absence of a temperature field, may be written as [2]:

$$M = \int_{\Gamma}\left(Wn_j - t_i\frac{\partial u_i}{\partial x_j}\right)x_j\,\mathrm{d}s \tag{2}$$

where n_j is the component of the unit outward normal on the contour Γ. W and t_j indicate the strain energy density and the traction component, respectively, and are given as $W = C_{ijkm}\,\varepsilon_{ij}\varepsilon_{km}/2$ and $t_j = \sigma_{ji}n_i$. In the presence of a temperature field, however, this form of the M-integral does not retain the property of path independence. In other words, it is not conserved for an arbitrary closed path for thermo-elastic deformations. Therefore, we rely upon the following modified form of the M-integral:

$$M = \int_{\Gamma_\varepsilon}(Wx_in_i - t_iu_{i,j}x_j)\mathrm{d}s \tag{3}$$

where Γ_ε is a vanishingly small contour of radius ε (see Fig. 2). Via the divergence theorem, one can straightforwardly show that Eq. (3) reduces to Eq. (2) in the absence of a body force and a temperature field.

Introducing the domain integral representation [11–13] and performing some manipulation, we can formulate the following expression:

$$M = \int_{A-A_\varepsilon}(\sigma_{li}u_{i,j} - W\delta_{jl})x_jq_{,l}\,\mathrm{d}A + \int_{A-A_\varepsilon}\alpha\sigma_{ll}(T_{,j}x_j + T)q\,\mathrm{d}A \tag{4}$$

where $W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^{\mathrm{m}}$, $\varepsilon_{ij}^{\mathrm{m}}$ is the mechanical strain without the thermal contribution, and $A - A_\varepsilon$ is the annular region surrounded by Γ_ε and the arbitrary path Γ, respectively. q is the weight function that is defined to be zero on Γ and 1 on Γ_ε with a linear variation between Γ and Γ_ε (see Fig. 2). The modified M-integral of Eq. (4) is then conserved for an arbitrary closed path Γ.

Suppose there are two independent elastic states 'A' and 'B'. We then consider another elastic state 'C' obtained by superposing the two equilibrium states 'A' and 'B'. Then the above M-integral is written as

$$M^{\mathrm{C}} = M^{\mathrm{A}} + M^{\mathrm{B}} + M^{(\mathrm{A,B})} \tag{5}$$

where the superscripts 'A', 'B' and 'C' indicate the aforementioned elastic states, and $M^{(\mathrm{A,B})}$ is the two-state M-integral, given as

$$M^{(\mathrm{A,B})} = \int_{A-A_\varepsilon}[\sigma_{li}^{A}u_{i,j}^{B} + \sigma_{li}^{B}u_{i,j}^{A} - \sigma_{pq}^{A}\varepsilon_{pq}^{B}]x_jq_{,l}\,\mathrm{d}A + \int_{A-A_\varepsilon}\alpha\sigma_{ll}^{B}(T_{,j}^{A}x_j + T^{\mathrm{A}})q\,\mathrm{d}A \tag{6}$$

The two-state M-integral $M^{(\mathrm{A,B})}$, derived from the M-integral, is associated with the mutual interaction between the two elastic states 'A' and 'B'.

3. The application of the two-state M-integral to a triple junction vertex

For applying the two-state M-integral to obtain the intensities of singularities at the vertex of the triple junction wedge as shown in Fig. 1, we briefly introduce the displacement and stress field in the form of an eigenfunction series.

The asymptotic eigenfunction solutions for the stress and displacement components of the present junction vertex may be written in the following power type eigenfunction of $z = x_1 + ix_2$ and $\bar{z} = x_1 - ix_2$ [8]:

$$\sigma_{\alpha\beta}^{(m)} = \mathrm{Re}\left[\sum_{\delta_n}\beta_n\sum_{k=1}^{2}[C_{kn}^{(m)}(\Lambda_{\alpha\beta k}g_n'(z) + \Gamma_{\alpha\beta k}\bar{z}g_n''(z)) + C_{(k+2)n}^{(m)}(\bar{\Lambda}_{\alpha\beta k}g_n'(\bar{z}) + \bar{\Gamma}_{\alpha\beta k}zg_n''(\bar{z}))]\right]$$

$$u_\alpha^{(m)} = \frac{1}{2\mu^{(m)}}\mathrm{Re}\left[\sum_{\delta_n}\beta_n\sum_{k=1}^{2}[C_{kn}^{(m)}(p_{\alpha k}^{(m)}g_n(z) + q_{\alpha k}\bar{z}g_n'(z)) + C_{(k+2)n}^{(m)}(\bar{p}_{\alpha k}^{(m)}g_n(\bar{z}) + \bar{q}_{\alpha k}zg_n'(\bar{z}))].\right] \quad (7)$$

with $g_n'(z) = z^{\delta_n}$, and the non-zero components of $\Lambda_{\alpha\beta k}$, $\Gamma_{\alpha\beta k}$, $p_{\alpha k}^{(m)}$, $q_{\alpha k}$:

$$-\Lambda_{111} = \Lambda_{221} = i\Lambda_{121} = 1, \quad -\Lambda_{112} = \Lambda_{222} = 2,$$
$$\Gamma_{112} = -\Gamma_{222} = -i\Gamma_{122} = -1, \quad p_{11}^{(m)} = -ip_{21}^{(m)} = -1,$$
$$p_{12}^{(m)} = ip_{22}^{(m)} = 3 - 4v^{(m)}, \quad q_{12} = -iq_{22} = -1$$

where δ_n is an eigenvalue and C_{kn}, short for $C_k(\delta_n)$, is the corresponding eigenvector; moreover, $\beta_n = \beta(\delta_n)$ represents the scaling load parameter or the intensity of the elastic field associated with the eigenvalue δ_n. Note that β_n is real for real δ_n, but in general, it is complex for a complex δ_n, in

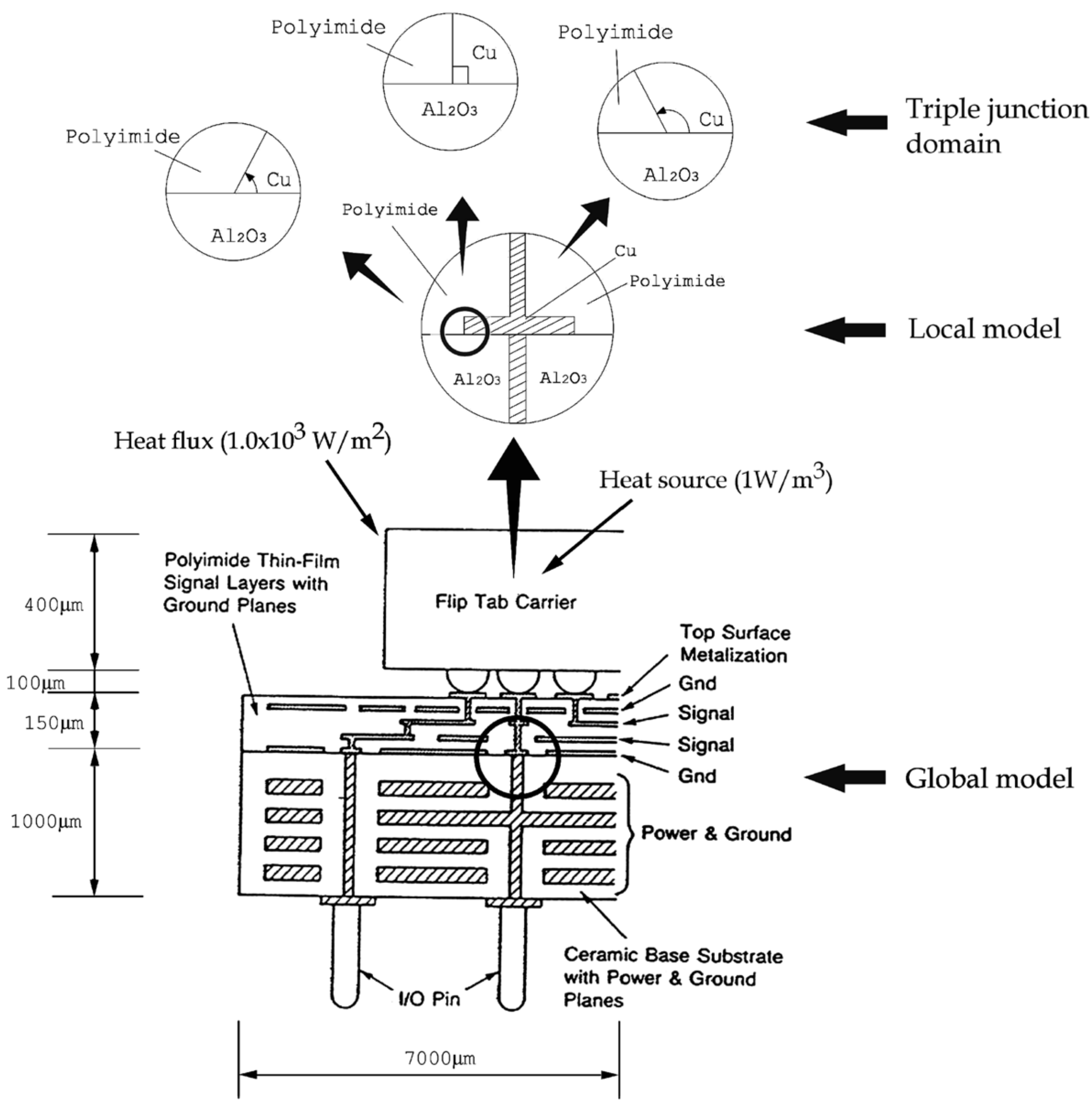

Fig. 3. The selected models in multilevel thin film package [14].

which case it is self-evident from the expression (7) that its conjugate $\bar{\delta}_n$ also belongs to the eigenvalues. For clarity we assume that the imaginary part of complex δ_n is positive in expression (7) since a complex eigenvalue δ_n and its conjugate $\bar{\delta}_n$ lead to the same eigenfunction. The superscript 'm' indicates the mth sector. We will omit the superscript 'm' for simplicity unless it is needed to distinguish one sector from another.

For the triple junction wedge under an elastic state, the bounded strain energy restricts the range of the eigenvalue δ_n such that $\mathrm{Re}(\delta_n) > -1$, and the terms giving rise to a singular stress field near the vertex are represented by the eigenvalues or the stress singularities δ_s within the range $-1 < \mathrm{Re}(\delta_s) < 0$.

Note that no temperature effects are included in the above expression; this will be discussed in the subsequence section. We now introduce an auxiliary field, which is to be superposed onto the elastic field under consideration in order to extract the intensity of the singular stress field near the vertex of the triple junction wedge.

Im and Kim [8] showed that for an eigenvalue δ_l there exists an auxiliary field in the form of a conjugate eigenfunction with the eigenvalue given by

$$\delta_l^c + \delta_l = -2 \tag{8}$$

For the stress singularity δ_s we have the conjugate eigenvalue $\delta_s^c = -2 - \delta_s$ and the auxiliary field, which is the elastic state corresponding to the eigenvalue δ_s^c and is given as

$$\begin{aligned} \sigma_{\alpha\beta}(\delta_s^c) &= \mathrm{Re}\left[\beta_s^c \sum_{k=1}^{2} [C_{ks}(\Lambda_{\alpha\beta k} g_s'(z) + \Gamma_{\alpha\beta k}\bar{z} g_s''(z)) \right. \\ &\quad \left. + C_{(k+2)s}(\bar{\Lambda}_{\alpha\beta k} g_s'(\bar{z}) + \bar{\Gamma}_{\alpha\beta k} z g_s''(\bar{z}))]\right] \\ u_\alpha(\delta_s^c) &= \frac{1}{2\mu}\mathrm{Re}\left[\beta_s^c \sum_{k=1}^{2} [C_{ks}(p_{\alpha k} g_s(z) + q_{\alpha k}\bar{z} g_s'(z)) \right. \\ &\quad \left. + C_{(k+2)s}(\bar{p}_{\alpha k} g_s(\bar{z}) + \bar{q}_{\alpha k} z g_s'(\bar{z}))]\right] \end{aligned} \tag{9}$$

with s being the index indicating the eigenvalue δ_s^c, so that

$$g_s(z) = z^{\delta_s^c} \quad \text{and} \quad C_{ks} = C(\delta_s^c)$$

where β_s^c is the scaling load parameter for the conjugate eigenfunction.

Now, we consider the superposition of the above auxiliary elastic state onto the given elastic state for the triple junction wedge. We substitute the elastic field (7) of the triple junction wedge under consideration for the elastic state 'A' and the conjugate field (9) for the elastic state 'B', which is an auxiliary field for Eq. (7). For the line integral path, we have chosen a circular arc or a contour with a vanishingly small radius ε as in Eq. (3). Then, after some algebra, the following expression for the two-state integral $M^{(\mathrm{A,B})}$ is found:

$$\begin{aligned} M^{(\mathrm{A,B})} = \sum_{n=1}^{\infty} \int_c & \mathrm{Re}[\varepsilon^{\delta_n+\delta_s^c+2}\beta_n\beta_s^c F(\delta_n, \delta_s^c, \theta) \\ & + \varepsilon^{\delta_n+\bar{\delta}_s^c+2}\beta_n\bar{\beta}_s^c G(\delta_n, \delta_s^c, \theta)]\mathrm{d}\theta \end{aligned} \tag{10}$$

where $F(\delta_n, \delta_s^c, \theta)$ and $G(\delta_n, \delta_s^c, \theta)$ are defined in each sector and given by Im and Kim [8]. We now rely upon expression (3) for $M^{(\mathrm{A,B})}$ in order to show that the contributions of all δ_n to the integral other than $\delta_n = -2 - \delta_s^c = \delta_s$ identically vanish; i.e. the integrand takes a positive power $(\mathrm{Re}(\delta_n + \delta_s^c + 2) > 0)$ of ε for $\delta_n \neq \delta_s$ or $\mathrm{Re}(\delta_n) > \mathrm{Re}(\delta_s)$ and it goes to zero as Γ_ε shrinks to zero. Let $\beta_s^c = \exp(\mathrm{i}\Psi)$ and $\beta_s = a_s - \mathrm{i}b_s$ where Ψ, a_s and b_s are real. Then, the above relation finally reduces to (see Im and Kim [8])

$$M^{(\mathrm{A,B})} = a_s I_a(\Psi, \delta_s) + b_s I_b(\Psi, \delta_s) \tag{11}$$

with

$$I_a(\Psi, \delta_s) = \int_0^{2\pi} \mathrm{Re}[\mathrm{e}^{\mathrm{i}\Psi} F(\delta_s, \delta_s^c, \theta) + \mathrm{e}^{-\mathrm{i}\Psi} G(\delta_s, \delta_s^c, \theta)]\mathrm{d}\theta$$

$$I_b(\Psi, \delta_s) = \int_0^{2\pi} \mathrm{Im}[\mathrm{e}^{\mathrm{i}\Psi} F(\delta_s, \delta_s^c, \theta) + \mathrm{e}^{-\mathrm{i}\Psi} G(\delta_s, \delta_s^c, \theta)]\mathrm{d}\theta$$

The integrals $I_a(\Psi, \delta_s)$ and $I_b(\Psi, \delta_s)$ may be accurately computed from Simpson's rule for a prescribed Ψ. We prescribe two different values for Ψ, most conveniently $\Psi = 0$ and $\Psi = \pi/2$, to compute the corresponding two sets of values for I_a and I_b. Then the above equation provides a set of two linear equations that determine the complex scaling load parameter $\beta_s = a_s - \mathrm{i}b_s$ associated with a singular eigenvalue δ_s if the two-state integrals corresponding to

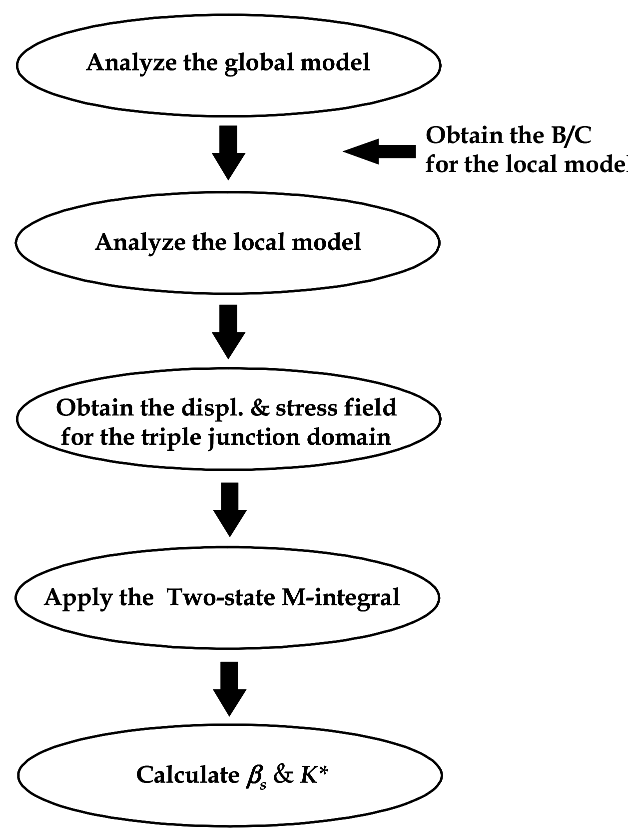

Fig. 4. The procedure of this research.

the two chosen values of Ψ are evaluated. Note that we only have to take $\Psi = 0$ for real δ_s since β_s is real, which is the case for the triple junction wedge under consideration. The accurate computation of the two-state integral $M^{(A,B)}$ is indeed possible via a regular displacement based FEM in conjunction with the domain integral representation in Eq. (6).

In this study we rely upon ABAQUS for computing the finite element solution. Then Eq. (11), with $\Psi = 0$, yields the intensity $\beta_s^c = a_s$ for the stress singularity δ_s in which $b_s = 0$. Note that the temperature term does not play a role in computing I_a and I_b in Eq. (6) because they are determined solely from the singular part of the near-field solution structure, which has nothing to do with the temperature field. The temperature plays a role in far-field loading and affects the unknown parameters β_s or a_s and b_s. This is why we do not pay attention to the temperature terms in the asymptotic eigenfunction expansion of Eq. (7).

4. Numerical examples

For this example, a multilevel thin film package as shown in Fig. 3 is considered [14]. During the operation of the chip heat is generated inside, part of the heat generated is convected out; however, the rest of the heat is conducted via the copper via, and should cause a singular stress near the triple junction vertex. For this analysis the global and local modeling technique is applied using ABAQUS. First, the whole multilevel thin film package is taken as a global model for the temperature and displacement analysis. To avoid the complexity of modeling, the effective models for the polyimide layer and Al_2O_3 substrate, which use the effective material properties recognizing the

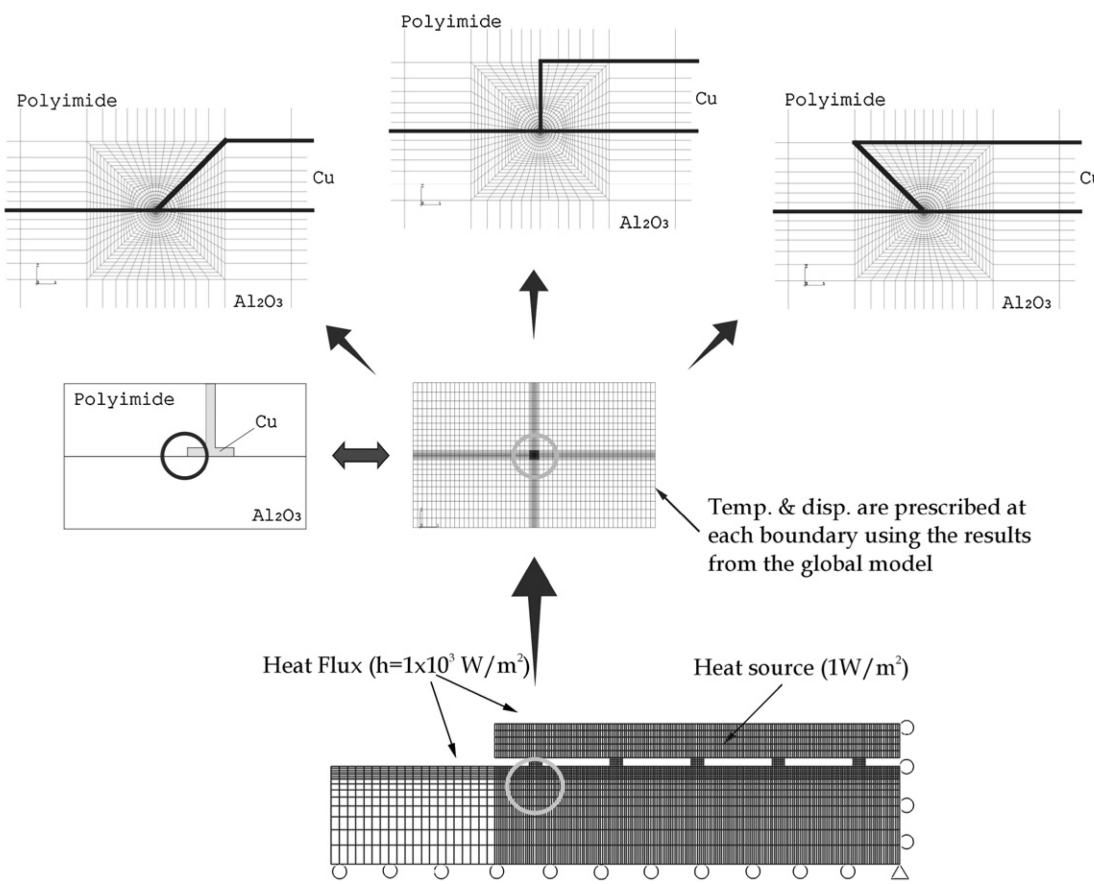

Fig. 5. The mesh configurations for the numerical analysis.

Table 1
Material properties for the numerical analysis

	Substrate	Polyimide	Solder	Chip	Cu
Young's modulus (GPa)	300.0e+09	100.0e+09	30.0e+09	188.0e+09	118.0e+09
Poison's ratio	0.25	0.35	0.3	0.26	0.34
CTE (ppm/°C)	6.0e−06	20.0e−06	28.0e−06	2.6e−06	16.6e−06
Conductivity (W/m°C)	25.0	5.0	36.0	150.0	401.0e+03

Table 2
Eigenvalues for each junction angle

$\alpha = 45°$	$\alpha = 90°$	$\alpha = 135°$
$1.99173 \pm i0.02031$	$1.99995 \pm i0.02076$	$2.00810 \pm i0.01982$
0.99989	$1.98952 \pm i0.12585$	1.94364
0.99917	$1.02614 \pm i0.02699$	$0.99986 \pm i0.01191$
0.02088	$9.72294 \pm i0.02556$	$0.99903 \pm i0.00826$
0.00856	0.02390	0.02247
0.0	0.0	0.0
−0.02239	−0.00039	−0.00941
−1.97761	−0.02470	−0.02106
−2.0	−1.97530	−1.97893
−2.00856	−1.99960	−1.99059
−2.02088	−2.0	−2.02247
−2.99917	−2.20390	$-2.99903 \pm i0.00826$
−2.99989	$-2.97229 \pm i0.02556$	$-2.99986 \pm i0.01191$
$-3.99173 \pm i0.02031$	$-3.02614 \pm i0.02699$	−3.94364

existence of the copper vias without considering their exact structure, are chosen. We choose the surrounding reference temperature to be 20 °C; the forced-air convection coefficient on the surface of the chip to be $h = 1.0 \times 10^3$ W/m^2; and the heat generation per unit cross section of the chip to be 1W/m^2. Then the temperature and the displacement distribution, which are extracted from the analysis of global model, are loaded along the boundary of the local model for analysis. For the local model, a simple structure of the copper via is included (see Fig. 3). Finally, the two-state M-integrals are calculated for the triple junction domains that consist of three different junction angles, $\alpha = 45°$, 90° and 135° in order to obtain the stress intensities (see Fig. 4).

Fig. 5 shows the mesh configurations of the multilevel thin film package as a global model, a small part around the perpendicular copper via (see Fig. 3) as a local model and the triple junction domains with three different junction angles, $\alpha = 45°$, 90° and 135° for numerical analysis. The size of the triple junction domain is chosen to be 20 μm × 20 μm. The material properties used for the numerical examples are shown in Table 1.

In Table 2, the eigenvalues δ_n are tabulated for each junction angle α. It was found that all stress singularities δ_s for $-1 < \mathrm{Re}(\delta_s) < 0$ are real. Moreover, all the eigenvalues exist as a conjugate pair, thereby satisfying relation (8). Fig. 6 shows the deformed shapes compared with the

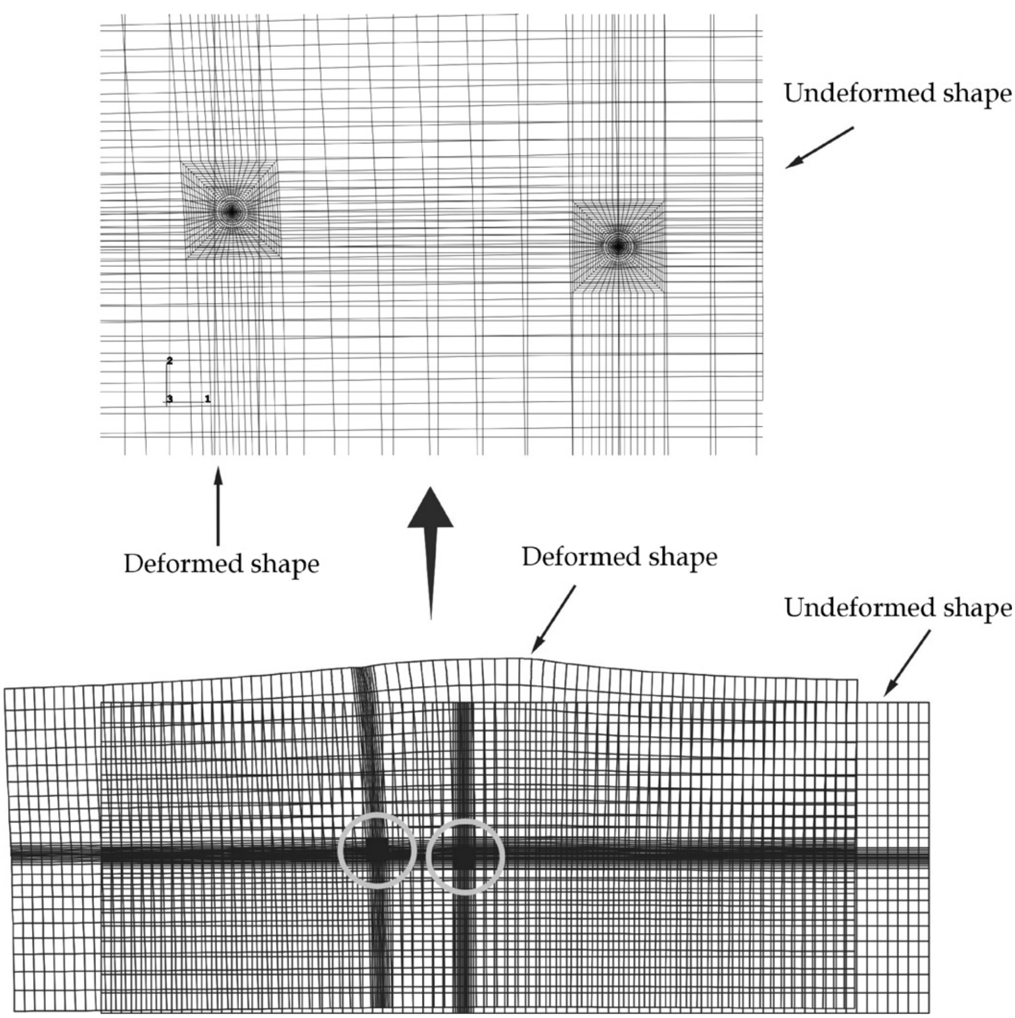

Fig. 6. The deformed shape of the local model and the triple junction domain (magnification factor = 180).

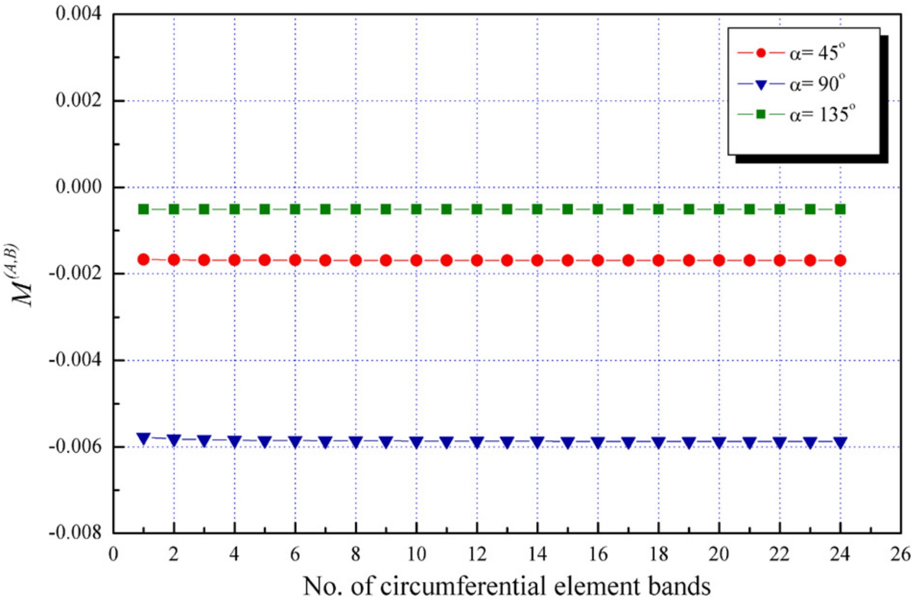

Fig. 7. The domain independence of two-state M-integral associated with three leading singularities of each junction angle.

Table 3
Results for dimensionless fracture parameter K^*

Junction angle, α	Stress singularities	Non-dimensionalized intensities
45°	−.2238735e−01	−.971823721e−03
90°	−.2469553e−01	−.79703275e−03
	−.3987466e−03	.139226023e−03
135°	−.2105816e−01	.22795964e−01
	−.9410821e−02	−.24917544e−04

undeformed shapes of the local model and the triple junction domain with $\alpha = 90°$. Fig. 7 shows the domain independence of expression (6) for the $M^{(A,B)}$ associated with the three leading singularities for each junction angle. The horizontal axis represents the total number of circumferential element bands in the region $A - A_\varepsilon$ of Fig. 2 that should increase according to the increase of Γ in the radial direction. Apparently, $M^{(A,B)}$ is independent of the number of the element bands.

The numerical results for β_s is now non-dimensionalized following Kim and Im [15]:

$$K^* = \frac{\sqrt{2\pi}\beta_s h^{\delta_s}}{E^*} \tag{12}$$

where E^* is the maximum Young's modulus of the three different materials and h is a typical dimension of the domain concerned. For this case it is the thickness of the copper. The normalized intensities K^* for each of the junction angles are now tabulated in Table 3.

5. Conclusion

The stress intensities for the triple junction vertex under thermal loading were calculated using the two-state M-integral at three different junction angles: $\alpha = 45°$, 90° and 135°. The present scheme gives a simple and efficient way to compute the intensities of the singular thermo-elastic stress field via a regular displacement based FEM without resorting to a singular tip element. As compared with the measured fracture toughnesses of the triple junction vertexes, the calculated normalized intensities can be used for designing reliable triple junction systems and therefore will be available to fabricate reliable multilevel thin film packages.

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References

[1] Chen FHK, Shield RT. Conservation laws in elasticity of the J-integral type. Z Angw Math Phys (ZAMP) 1977;28:1–22.
[2] Knowles JK, Sternberg E. On a class of conservation laws in a linearized and finite elastostatics. Arch Rat Mech Anal 1978;44:187–211.
[3] Yau JF, Wang SS, Corten HT. A mixed mode crack analysis of isotropic solids using conservation laws of elasticity. ASME J Appl Mech 1980;47:335–41.
[4] Kfouri AP. Some evaluation of elastic T-term using Eshelby's method. Int J Fract 1986;30:301–15.
[5] Shih CF, Asaro RJ. Elastic–plastic analysis of cracks on bimaterial interfaces: part I – small scale yielding. ASME J Appl Mech 1988;55:299–316.
[6] Jeon I, Im S. The role of higher order eigenfields in elastic–plastic cracks. J Mech Phys Solids 2001;49:2789–818.
[7] Choi NY, Earmme YY. Evaluation of stress intensity factors in a circular arc-shaped interfacial crack using L-integral. Mech Mater 1992;14:141–53.

[8] Im S, Kim SK. Application of the two-state M-integral for computing an intensity of a singular near-tip field for a generic composite wedge. J Mech Phys Solids 2000;48:129–51.
[9] Lee Y, Im S. On the computation of the near-tip stress intensity for three-dimensional wedges via two-state M-integral. J Mech Phys Solids 2003;51:825–50.
[10] Lee Y, Jeon I, Im S. The stress intensities of three-dimensional corner singularities in a laminated composite. Int J Solid Struct 2006;43:2710–22.
[11] Kishimoto K, Aoki S, Sakata M. On the path independent integral $\widehat{J}$. Eng Fract Mech 1980;13:841–50.
[12] Li FZ, Shih CF, Needleman A. A comparison of methods for calculating energy release rates. Eng Fract Mech 1985;21:401–21.
[13] Moran B, Shih CF. Crack tip and associated domain integral from momentum and energy balance. Eng Fract Mech 1987;27:615–42.
[14] Tummular RR. Microelectronics packaging handbook. New York: Van Nostrand Reinhold; 1989.
[15] Kim TW, Im S. Boundary layers in wedges of laminated composite strip under generalized plane deformation – part I and part II. Int J Solid Struct 1995;32:609–45.