Rayleigh surface wave in a piezoelectric wafer with subsurface damage

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(Received 31 October 2009; accepted 2 December 2009; published online 30 December 2009)

An analytical study is carried out on the propagation of Rayleigh surface waves in a piezoelectric wafer with subsurface damage. The region of subsurface damage is considered to be a functionally graded piezoelectric thin film. The findings show the influence of the gradient parameter, thickness of the region of subsurface damage, and three different types of damage on the properties of surface-wave propagation, including the phase velocity and electromechanical coupling factor. They can provide theoretical guidance in nondestructive evaluation for the analysis of the reliability and durability of electronic devices made of piezoelectric wafers.
[doi:10.1063/1.3276568]

Due to the electromechanical coupled characteristics, piezoelectric wafers are widely used for electronic devices such as sensors,[1] actuators,[2] surface acoustic wave devices,[3] microelectromechanical system,[4] and so on. However, the process of wafer machining, the corrosive environment, and fatigue under periodic mechanical or thermal loading during device operation will lead to subsurface damage in the piezoelectric wafer. These affect the elastic, piezoelectric, and dielectric properties in the subsurface of the wafer. To achieve high performance of devices, an investigation of the effects of subsurface damage on piezoelectric wafers is necessary.

Many studies have been carried out on ultrasonic, nondestructive evaluation for surface properties using Rayleigh surface-waves.[5,6] Most of these studies have assumed the subsurface material to be an elastic medium[5–7] and made the simplifying assumption that the wafer with subsurface damage is a multilayered structure[6] that has homogenous but discontinuous material properties. Then, the dispersion characteristics of the surface layers were measured by the stepwise removal of each surface layer. Many papers have been published on transverse surface-waves in inhomogeneous half-spaces[8] or layered structures.[9] However, no theoretical results have been published on the propagation of Rayleigh surface-waves in a structure that is covered by an inhomogeneous thin film of which the properties vary continuously along the depth.

In this letter, we assume that the subsurface damage layer shows continuous material properties along its depth. Hence, we consider it to be a functionally graded piezoelectric material (FGPM) thin film that covers the homogenous substrate. The Rayleigh surface-wave that propagates along the piezoelectric wafer with subsurface damage is taken into account, as shown in Fig. 1. Here, the substrate is treated as a half-space because the thickness of the substrate is typically much larger than that of the subsurface damage layer, h. It is assumed that the xoy coordinate plane is an isotropic plane; the direction of polling is the same as the positive direction of the z-axis; and Rayleigh waves propagate along the positive direction of the x-axis. Thus, the displacement vector and the electrical potential are given by the following: $u=u(x,z,t)$, $v=0$, $w=w(x,z,t)$, and $\varphi=\varphi(x,z,t)$.

Let u_1, w_1, and φ_1 denote the in-plane displacement, outplane displacement, and electrical potential in the FGPM layer (hereafter, $\exp[ik(x-ct)]$ is omitted for brevity), respectively. Then, the governing equations for the propagation of the Rayleigh surface-wave in the layer can be expressed as follows:[10]

$$\hat{c}_{44}u_1''+\hat{c}_{44}'u_1'+(\hat{\rho}c^2-\hat{c}_{11})k^2u_1+[(\hat{c}_{13}+\hat{c}_{44})ikw_1' + (\hat{e}_{31}+\hat{e}_{15})ik\varphi_1'+\hat{c}_{44}'ikw_1+\hat{e}_{15}'ik\varphi_1]=0,$$

$$\hat{c}_{33}w_1''+\hat{c}_{33}'w_1'+(\hat{\rho}c^2-\hat{c}_{44})k^2w_1+[(\hat{c}_{13}+\hat{c}_{44})iku_1'+\hat{e}_{33}\varphi_1'' - \hat{e}_{15}k^2\varphi_1+\hat{c}_{13}'iku_1+\hat{e}_{33}'\varphi_1']=0,$$

$$\hat{\varepsilon}_{33}\varphi_1''+\hat{\varepsilon}_{33}'\varphi_1'-\hat{\varepsilon}_{11}k^2\varphi_1+[\hat{e}_{15}k^2w_1-(\hat{e}_{31}+\hat{e}_{15})iku_1' - \hat{e}_{33}w_1''-\hat{e}_{31}'iku_1-\hat{e}_{33}'w_1']=0, \tag{1}$$

where c_{ijkl}, e_{kij}, and ε_{jk} are the elastic, piezoelectric, and dielectric coefficients, respectively, and the symbol, "^", denotes the material parameters in the FGPM layer.

Similarly, let u_2, w_2, and φ_2 denote displacement components and electrical potential in the piezoelectric substrate, respectively. Let φ_0 denote the electrical potential in the air. The governing equations in the homogenous substrate and the air have been reported in Reference 13. Additionally, the following boundary conditions, interface continuity conditions, and attenuation conditions should be satisfied, including two kinds of electrical boundary conditions, i.e., electri-

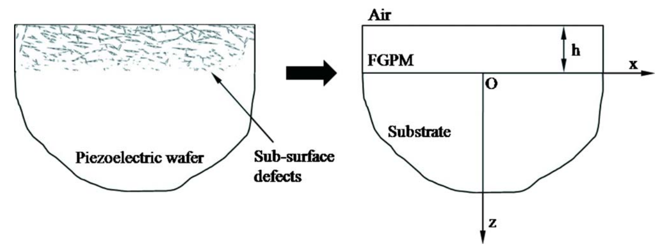

FIG. 1. (Color online) The schematic diagram of piezoelectric material with subsurface damage.

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cal open and short conditions. (1) The mechanical traction free condition at $z=-h: \sigma_z=0$ and $\tau_{xz}=0$. (2) The electrical boundary condition at $z=-h$; (a) electrical short case, $\varphi_1=0$ and (b) electrical open case, $\varphi_1=\varphi_0$ and $D_{z1}=D_{z0}$. (3) The continuous conditions at $z=0$: $u_1=u_2$, $w_1=w_2$, $\varphi_1=\varphi_2$, $\sigma_{z1}=\sigma_{z2}$, $\tau_{xz1}=\tau_{xz2}$, and $D_{z1}=D_{z2}$. (4) The attenuation conditions: for $z\to\infty$, u_2, w_2, and $\varphi_2\to 0$ and for $z\to-\infty$, $\varphi_0\to 0$.

To solve the variable-coefficient ordinary differential equations, viz., Eq. (1), both the material parameters and solutions can be expressed as power series regarding $(-z/h)^n$ as follows:[11,12]

$$M_m=\sum_{n=0}^{\infty}\alpha_n^m\left(\frac{z}{-h}\right)^n,\quad u_1=\sum_{n=0}^{\infty}s_n\left(\frac{z}{-h}\right)^n,$$

$$w_1=i\sum_{n=0}^{\infty}t_n\left(\frac{z}{-h}\right)^n,\quad \varphi_1=i\sum_{n=0}^{\infty}r_n\left(\frac{z}{-h}\right)^n, \tag{2}$$

where M_m $(m=1-10)$ denote $\hat{c}_{11}$, $\hat{c}_{13}$, $\hat{c}_{33}$, $\hat{c}_{44}$, $\hat{e}_{15}$, $\hat{e}_{33}$, $\hat{e}_{31}$, $\hat{\varepsilon}_{11}$, $\hat{\varepsilon}_{33}$, and $\hat{\rho}$, respectively, and the coefficients $\{\alpha_n^m\}$ can be determined by the relationships between the functions and their Taylor expansions.

Substituting Eqs. (2) into Eqs. (1) and equating the coefficients of $(-z/h)^n$, we obtain a series of recursive equations for s_n, t_n, and r_n. Through these equations, s_n, t_n, and r_n for $n>2$ can be solved as linear functions of undetermined constants, viz., s_0, t_0, r_0, s_1, t_1, and r_1. To decouple the undetermined coefficients, the solution can also be expressed as[12]

$$u_1=\sum_{j=1}^{4}C_j\sum_{n=0}^{\infty}s_{nj}\left(\frac{z}{-h}\right)^n,$$

$$w_1=i\sum_{j=1}^{4}C_j\sum_{n=0}^{\infty}t_{nj}\left(\frac{z}{-h}\right)^n,$$

$$\varphi_1=i\sum_{j=1}^{4}C_j\sum_{n=0}^{\infty}r_{nj}\left(\frac{z}{-h}\right)^n, \tag{3}$$

where $(s_{0j},s_{1j},t_{0j},t_{1j},r_{0j},r_{1j})=I$; in the above, $j=1-6$ and I is a 6×6 unit matrix. Hence, $\{C_j: j=1,\ldots,6\}$ are undetermined constants.

The displacement components, the electrical potential in the homogenous substrate, and the electric potential in the air[13] are

$$u_2=C_7e^{\lambda_1kz}+C_8e^{\lambda_2kz}+C_9e^{\lambda_3kz},$$

$$w_2=i(C_7l_{11}e^{\lambda_1kz}+C_8l_{12}e^{\lambda_2kz}+C_9l_{13}e^{\lambda_3kz}),$$

$$\varphi_2=i(C_7l_{21}e^{\lambda_1kz}+C_8l_{22}e^{\lambda_2kz}+C_9l_{23}e^{\lambda_3kz}),$$

$$\varphi_0=iC_{10}e^{kz}, \tag{4}$$

where $\{C_j: j=7,\ldots,10\}$ are undetermined constants.

Substituting Eqs. (3) and (4) into the boundary and continuity conditions for the electrical open case, we obtain a set of homogeneous linear algebraic equations for determining $C_i(i=1-10)$ while for the electrical short case, C_{10} is superfluous.

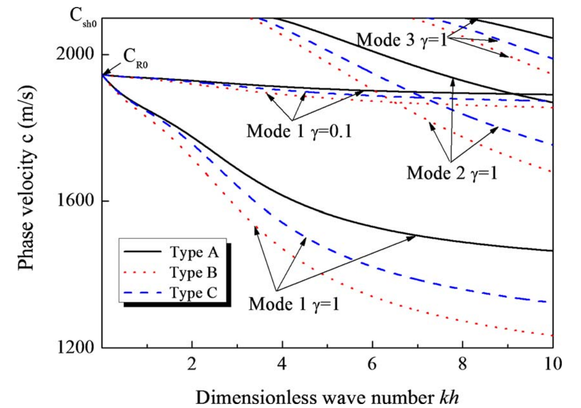

FIG. 2. (Color online) Phase velocity c of Rayleigh waves in a PZT-1 half space with subsurface damage plotted as a function of dimensionless wave number kh.

Actually, various factors cause different types of surface damage. Given that the variation in the density always is so slight, we suppose the density to be a constant. The elastic properties, piezoelectric properties, and dielectric properties might vary along the thickness individually or together. Therefore, we select three different possible damage types, viz., types A, B, and C that, respectively, indicate that the following properties vary along the depth: only mechanical properties; both mechanical and piezoelectric properties; and all properties except the density. Suppose that the gradient function is as follows:

$$g=\exp(\gamma z/h), \tag{5}$$

where γ is the gradient parameter. We select PZT-1 as the wafer and the material parameters are shown as follows: $c_{11}=135$ GPa, $c_{13}=67.9$ GPa, $c_{44}=22.2$ GPa, $c_{33}=113$ GPa, $e_{15}=9.8$ C/m^2, $e_{33}=9.0$ C/m^2, $e_{31}=-1.9$ C/m^2, $\varepsilon_{11}=990\varepsilon_0$, $\varepsilon_{33}=450\varepsilon_0$, $\varepsilon_0=8.854\times10^{-12}$ F/m, and $\rho=7.5\times10^3$ kg/m^3. When the parameters vary along thickness, they follow $\hat{p}=pg$, where $\hat{p}$ denote each parameter in FGPM layer, p is that in substrate listed above and g is defined by Eq. (5).

Figure 2 shows the dispersion curves of the surface wave that propagates in the layered structure, where c_{R0} and c_{sh0}, respectively, denote the phase velocity of Rayleigh waves and bulk shear waves in homogenous material. Normally, in infinite and homogenous half-space, a Rayleigh surface-wave propagates in only one mode and the phase velocity is a constant, which is called nondispersion. Given that the group velocity is $c_g=c+kdc/dk$, we note that subsurface damage leads to the following: a decrease in the phase velocity as the wave number increases; normal dispersion because $c_g<c$, and different modes. We define the limit on the phase velocity in the first mode as $\bar{c}$, which depends on the gradient parameter γ, as shown in Fig. 3. For a certain type, when γ varies from 0 to 1, the value of $\bar{c}$ decreases almost linearly when γ increases. However, the influence of the gradient parameter γ on the value of $\bar{c}$ varies with the nature of variation of material properties. The value of $\bar{c}$ falls the fastest when γ increases in type B. That is, if both the elastic and piezoelectric parameters vary along the depth, the effect of γ is the greatest. In type A, where only the elastic property varies along the depth, the effect of γ is the smallest. Due to the dispersion curves in Fig. 2 that describe the relationship between the phase velocity c, and the dimensionless wave

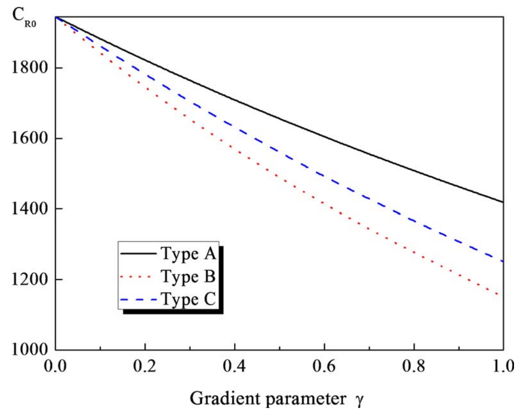

FIG. 3. (Color online) The limitation of phase velocity $\bar{c}$ of Rayleigh waves in a PZT-1 half space with subsurface damage plotted as a function of gradient parameter γ.

number kh, the thickness of the region of subsurface damage can be calculated by the relationship between the phase velocity c, and the frequency, ω. The greater is the thickness, the faster the phase velocity decreases as the frequency increases.

In addition, the material property vary along depth might lead to an increase in the number of modes. The greater the gradient parameter is, the more the modes are. For example, when the dimensionless wave number varies from 0 to 10, there is only one mode at $\gamma=0.1$ but three modes at $\gamma=1$, while there is only one mode without dispersion for Rayleigh wave propagating in homogenous piezoelectric half space and independent on wave number. The reason is that a greater gradient parameter implies a greater difference in the shear wave velocity between the upper surface and the substrate. Furthermore, the type of damage also affects the dispersion curves, as shown in Fig. 2. The effect of type B is the most obvious, that of type C comes next, and that of type A is the least obvious.

The electromechanical coupling factor for surface waves can be defined as $\kappa^2=2[(c_o-c_s)/c_o]$, where c_o and c_s, respectively, are the phase velocity for the electrical open and electrical short cases. Figure 4 illustrates the coupled electromechanical factors of the surface wave that propagates along

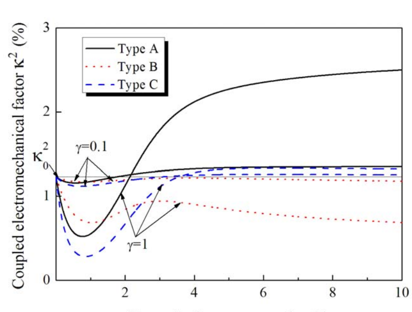

FIG. 4. (Color online) The coupled electromechanical factor κ^2 of Rayleigh waves in a PZT-1 half space with subsurface damage plotted as a function of dimensionless wave number kh.

the piezoelectric media with surface damage, where κ_0^2 represents the electromechanical coupling factor in the homogenous, piezoelectric half-space. We note that not only the gradient parameter γ but also the damage types affect the coupled electromechanical factors. If only the elastic parameter varies along the depth, the electromechanical coupling factor in the FGPM layered structure is larger than that in homogenous, piezoelectric half-space when the dimensionless wave number kh, is larger than about 2. If both elastic and piezoelectric properties vary along the depth, the coupled electromechanical factor is less than that in homogenous piezoelectric half-space. If all the properties except the density vary along the depth, the electromechanical factor decreases sharply and attains the minimum when the dimensionless wave number kh, is about 0.8. The electromechanical factor increases and becomes smooth when kh is larger than about 3. In other words, when all the properties except the density vary along the depth, the coupled electromechanical factor decreases at small wave numbers and slightly increases at large wave numbers.

The Rayleigh surface-wave was used to analyze the effects of subsurface damage on piezoelectric wafers. The obtained results show that both the gradient parameter and the type of damage affect the propagation of surface waves. The limit on the phase velocity in the first mode is closely dependent on the gradient parameter. A larger thickness of the layer of subsurface damage can cause the phase velocity to fall faster as the frequency increases. Furthermore, although the shapes of the dispersion curves of various types of damage are almost similar, the difference in the coupled electromechanical factor is obvious. All the results should yield theoretical guidance for nondestructive evaluation for reliability analysis and the prediction of durability of electronic devices that are made by piezoelectric wafers.

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (Grant No. KRF-2008-331-D00002).

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